

# Lecture 5: Different models for vaccine mechanism

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## SURVIVAL ANALYSIS IN A NUTSHELL

$T$  - random variable for time to the event

PDF:  $f(t) = \lim_{dt \rightarrow 0} P[t < T \leq t + dt]/dt$

CDF:  $F(t) = P[T \leq t]$

Survival function:  $S(t) = P[T > t] = 1 - F(t)$

Hazard function:  $\lambda(t) = \frac{f(t)}{S(t)}$

Integrated hazard function:  $\Lambda(t) = \int_0^t \lambda(\tau) d\tau$

$$S(t) = e^{-\Lambda(t)}$$

$$F(t) = AR(t) = 1 - S(t)$$

FOR AN INFECTIOUS DISEASE

$$\lambda(t) = cp \frac{I(t)}{n}$$

SIR EPIDEMIC

$$\frac{dS(t)}{dt} = -cp \frac{I(t)}{n} S(t) = -\lambda(t)S(t)$$

$$\frac{dI(t)}{dt} = cp \frac{I(t)}{n} S(t) - \gamma I(t)$$

$$\frac{dR(t)}{dt} = \gamma I(t),$$

$$S(t) + I(t) + R(t) = n, \text{ for all } t,$$

$$S(0) = n - a, I(0) = a > 0, R(0) = 0.$$

$$\frac{dI(t)}{dt} = \left[ \frac{cp S(t)}{\gamma n} - 1 \right] \gamma I(t)$$

**Basic Reproductive Number:**

$$R_0 = \frac{cp}{\gamma}$$

## DYNAMICS OF SIR EPIDEMIC

$$\frac{dI(t)}{dt} = \left[ R_0 \frac{S(t)}{n} - 1 \right] \gamma I(t)$$

if  $R_0 \frac{S(t)}{n} \leq 1$ , then  $\frac{dI(t)}{dt} \leq 0$

if  $R_0 \frac{S(t)}{n} > 1$ , then  $\frac{dI(t)}{dt} > 0$

near  $t = 0$ ,  $\frac{S(t)}{n} \approx 1$ , and

$$\frac{dI(t)}{dt} \Big|_{t=0^+} = [R_0 - 1] \gamma I(t)$$

if  $R_0 \leq 1$ , then no epidemic occurs

if  $R_0 > 1$ , then an epidemic occurs

## SURVIVAL FUCTION

Let  $a = 0^+$  and  $S(0) = n^-$

$$\frac{dS(t)}{dt} = -\lambda(t)S(t)$$

solving yields

$$\frac{S(t)}{n} = e^{-\Lambda(t)}$$

where

$$\Lambda(t) = \frac{cp}{n} \int_0^t I(\tau) d\tau$$

Let  $AR(t) = 1 - \frac{S(t)}{n}$ , then

$$AR(t) = 1 - e^{-\Lambda(t)}$$

## THE VACCINE MODEL

Force of infection to an unvaccinated person

$$\lambda_0(t) = Z_0 c \pi p(t)$$

$$\text{where } p(t) = \left[ \frac{n_0 p_0(t) + n_1 \phi p_1(t)}{n} \right] .$$

and to a vaccinated person,

$$\lambda_1(t) = Z_1 \theta c \pi p(t) .$$

$$S_v(t) = E\{\exp[-Z_v \Lambda_v(t)]\} = L_{Z_v} [\Lambda_v(t)] .$$

$$\text{where } \Lambda_0(t) = c \pi \int_0^t p(\tau) d\tau \text{ and } \Lambda_1(t) = c \pi \theta \int_0^t p(\tau) d\tau .$$

## MIXING MODEL

$$P(Z_v = 0) = \alpha_v,$$

and

$$Z_v | Z_v > 0 \sim f_v(\cdot), \text{ with probability } 1 - \alpha_v,$$

where  $E(X_v) = 1$  and  $\text{Var}(X_v) = \delta_v$

$$E(Z_v) = 1 - \alpha_v \text{ and } \text{Var}(Z_v) = (1 - \alpha_v)(\delta_v + \alpha_v)$$

$$L_{Z_v}(s) = \alpha_v + (1 - \alpha_v)L_{X_v}(s).$$



## MIXING MODEL (GAMMA DISTRIBUTION)

$X_v$  gamma with scale and shape parameters  $1/\delta_v$ .

$$L_{Z_v}(s) = \alpha_v + (1 - \alpha_v) \left[ \frac{1}{1 + s\delta_v} \right]^{1/\delta_v}$$

$$S_v(t) = \alpha_v + (1 - \alpha_v) \left[ \frac{1}{1 + \Lambda_v(t)\delta_v} \right]^{1/\delta_v}$$

When  $\alpha_v = 0$ ,

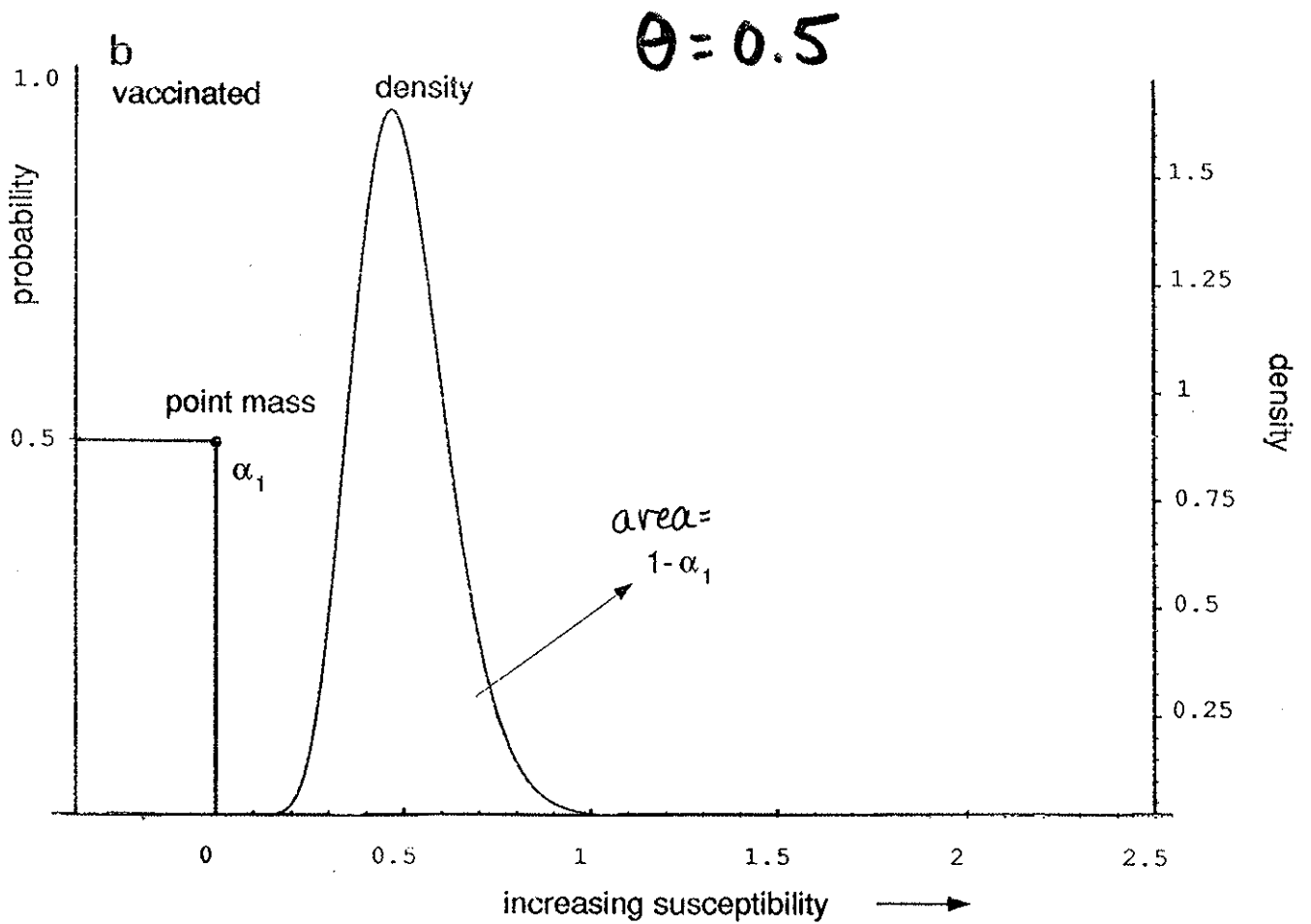
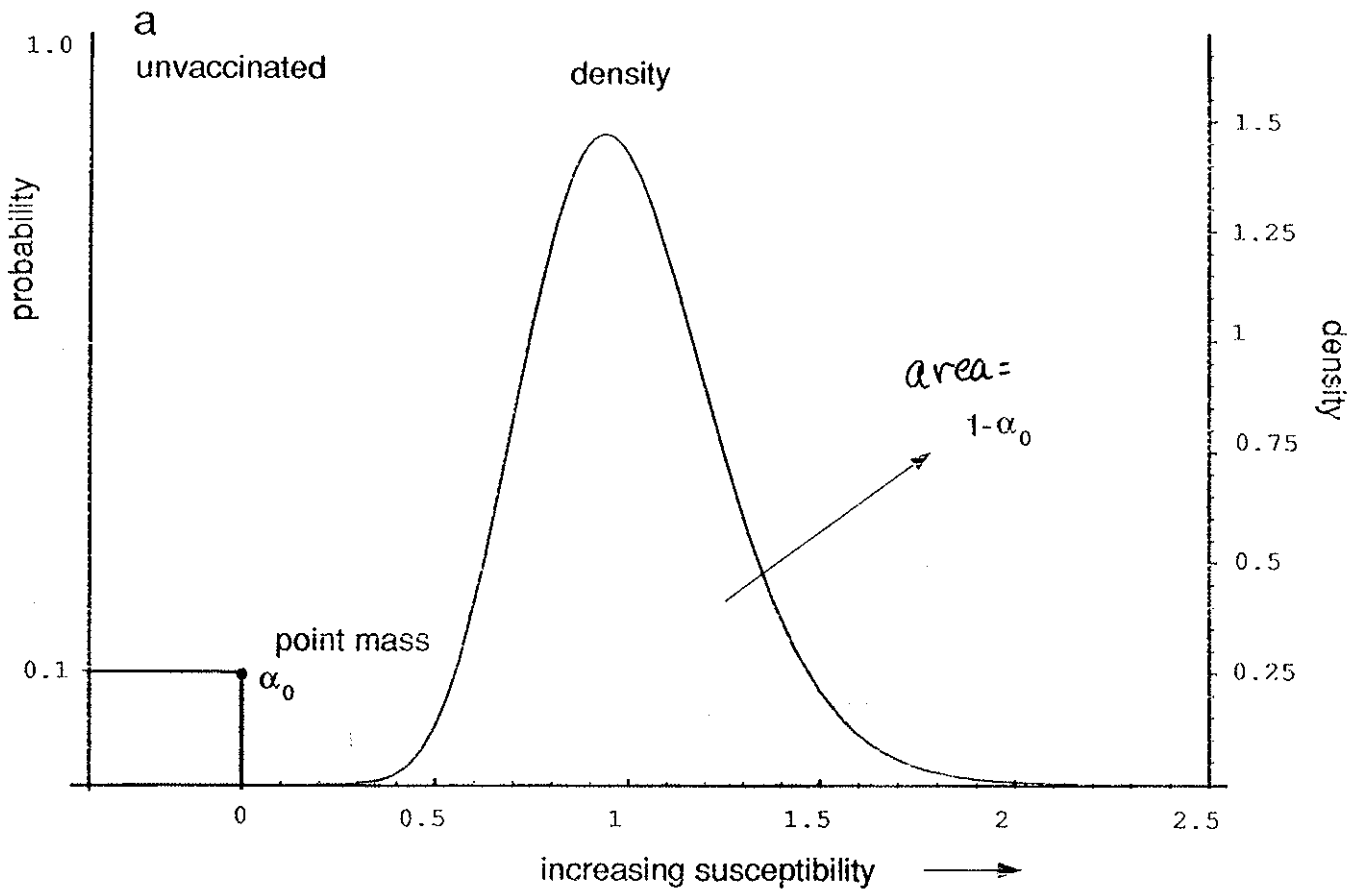
$$S_v(t) = \left[ \frac{1}{1 + \Lambda_v(t)\delta_v} \right]^{1/\delta_v}$$

When  $\delta_v = 0$ ,

$$S_v(t) = \alpha_v + (1 - \alpha_v) \exp[-\Lambda_v(t)]$$

$$S_0(t) = \alpha_0 + (1 - \alpha_0) \exp[-\Lambda_0(t)]$$

$$S_1(t) = \alpha_1 + (1 - \alpha_1) \exp[-\theta \Lambda_0(t)]$$



MODEL OF AALEN (COMPOUND POISSON)  
(Annals of Applied Probability, 1992)

$$L_{Z_v}(s) = \exp \left\{ \frac{\alpha_v}{(1 - \alpha_v)\delta_v} \left[ 1 - \{1 + (s\delta_v/\alpha_v)\}^{1-\alpha_v} \right] \right\}, \quad \alpha_v \neq 1, \quad \alpha_v > 0$$

$$P(Z_v = 0) = \exp \left\{ \frac{\alpha_v}{(1 - \alpha_v)\delta_v} \right\}$$

$$S_v(t) = \exp \left\{ \frac{\alpha_v}{(1 - \alpha_v)\delta_v} \left[ 1 - \{1 + (\Lambda_v(t)\delta_v/\alpha_v)\}^{1-\alpha_v} \right] \right\}$$

When  $\alpha_v = 1$ ,

$$S_v(t) = \left[ \frac{1}{1 + \Lambda_v(t)\delta_v} \right]^{1/\delta_v}.$$

## VACCINE EFFICACY

$$VE_S = 1 - \frac{(1 - \alpha_1)\theta\pi}{(1 - \alpha_0)\pi} = 1 - \frac{(1 - \alpha_1)}{(1 - \alpha_0)} \theta$$

**Special cases:**

$$\alpha_0 = 0, \quad VE_S = 1 - (1 - \alpha_1) \theta$$

$$\alpha_0 = \alpha_1, \quad VE_S = 1 - \theta \quad \text{"leaky"}$$

$$\alpha_0 = 0, \quad \theta = 1, \quad VE_S = \alpha_1 \quad \text{"all-or-none"}$$

Halloran, *et al.* (1992)

$$VE_I = 1 - \phi$$

RELATIONSHIPS AMONG MODEL PARAMETERS AND VE MODELS

Model name	$\alpha_0$	$\alpha_1$	$\theta$	VE
General	$\alpha_0 > 0$	$\alpha_1 > 0$	$\theta \neq 1$	$1 - \frac{(1 - \alpha_1)}{(1 - \alpha_0)} \theta$
Relative all-or-none	$\alpha_0 > 0$	$\alpha_1 > 0$	$\theta = 1$	$\frac{\alpha_1 - \alpha_0}{1 - \alpha_0}$
All-or-none*	$\alpha_0 = 0$	$\alpha_1 > 0$	$\theta = 1$	$\alpha_1$
All-or-partially susceptible*	$\alpha_0 = 0$	$\alpha_1 > 0$	$\theta \neq 1$	$1 - (1 - \alpha_1)\theta$
Partially susceptible*	$\alpha_0 = \alpha_1$		$\theta \neq 1$	$1 - \theta$
Risk difference all-or-none**	$\alpha_0 > 0$	$\alpha_1 > 0$	$\theta = 1$	$\alpha_1 - \alpha_0$

\* Model previously described by Halloran, et al. (1992). Note that "leaky" has been changed to "partially susceptible."

\*\* Model not contained in the general model.

ESTIMATING VE FROM FINAL VALUE DATA  
PARTIALLY SUSCEPTIBLE CASE ( $\delta_0 = \delta_1 = 0, \alpha_0 = \alpha_1 = 0$ )

$$AR_v(t) = 1 - S_v(t).$$

$$\ln[1 - AR_0(t)] = -\Lambda_0(t)$$

$$\ln[1 - AR_1(t)] = -\theta\Lambda_0(t)$$

$$\theta = \ln[1 - AR_1(t)] / \ln[1 - AR_0(t)]$$

$$VE_S = 1 - \theta$$

$$\lambda_1(t) = \theta \lambda_0(t)$$

$$\theta = e^{\beta}$$

$$\hat{VE} = 1 - \frac{\hat{AR}_1}{\hat{AR}_0} \quad \text{PARTIALLY SUSCEPTIBLE CASE (CONTINUED)}$$

$$\hat{VE}_S = 1 - [\ln(1 - \hat{AR}_1(t)) / \ln(1 - \hat{AR}_0(t))] \geq 1 - \frac{\hat{AR}_1}{\hat{AR}_0}$$

Note that

$$\text{Var}[\hat{AR}_v(t)] \cong AR_v(t) [1 - AR_v(t)] / n_v,$$

and that  $AR_0(t)$  and  $AR_1(t)$  are conditionally independent.

Then use the delta method to yield

$$\text{Var}[\hat{VE}] \cong [AR_1(t)/n_1 + \theta^2 AR_0(t)/n_0] [\log_e(1 - AR_0(t))]^{-2} \quad (\text{Becker, 1982})$$

**Regression model:**  $[1 - AR_1(t)] = [1 - AR_0(t)]^\theta$  proportional hazards model

ALL-OR-NONE ( $\delta_0 = \delta_1 = 0$ ,  $\alpha_0 = 0$ ,  $\alpha_1 = \alpha$ ,  $\theta = 1$ )

$$[1 - AR_0(t)] = \exp\{-\Lambda_0(t)\}$$

$$[1 - AR_1(t)] = \alpha + (1 - \alpha) \exp\{-\Lambda_0(t)\} = \alpha + (1 - \alpha)[1 - AR_0(t)]$$

$$AR_1(t) = (1 - \alpha) AR_0(t)$$

$$\alpha = 1 - [AR_1(t)/AR_0(t)]$$



ALL-OR-NONE (CONTINUED)

$$\text{Variance of } \hat{\alpha} = 1 - [\hat{AR}_1(t)/\hat{AR}_0(t)]$$

Let  $a = \log_e(1 - \alpha)$ , then

$$\hat{a} = \ln[\hat{AR}_1(t)] - \ln[\hat{AR}_0(t)].$$

Then by the delta method,

$$\text{Var}[\hat{a}] = [(1 - AR_1(t))/(n_1 AR_1(t))] + [(1 - AR_0(t))/(n_0 AR_0(t))]$$

(O'Neill, 1988)

**Regression Model:**  $AR_0(t) = [1 + \exp(b_0)]^{-1}$ ,  $AR_1(t) = [1 + \exp(b_0 + b_1)]^{-1}$ .

$$\hat{VE} = 1 - \frac{1 + \exp(\hat{b}_0)}{1 + \exp(\hat{b}_0 + \hat{b}_1)}$$

$$ROR = e^{b_1}$$

$$\hat{VE} \neq 1 - e^{\hat{b}_1}$$

# LOG-LOG PLOTS

Plot  $\ln\{-\ln[S_p(t)]\}$  vs.  $t$

e.g., pure leaky model ( $\delta_0 = \delta_1 = 0, \alpha_0 = \alpha_1 = 0$ )

$$S_0(t) = \exp[-\Lambda_0(t)]$$

$$S_1(t) = \exp[-\theta\Lambda_0(t)]$$

$$-\ln[S_0(t)] = \Lambda_0(t)$$

$$-\ln[S_1(t)] = \theta\Lambda_0(t)$$

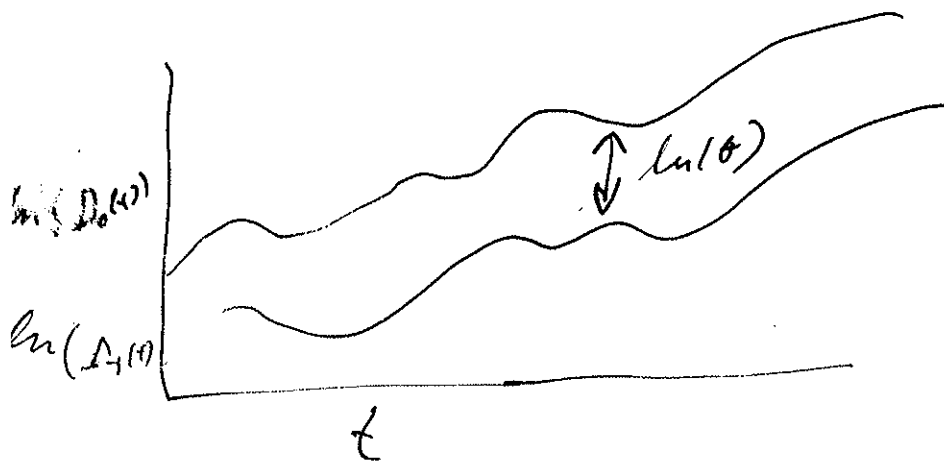
$$\ln\{-\ln[S_0(t)]\} = \ln[\Lambda_0(t)]$$

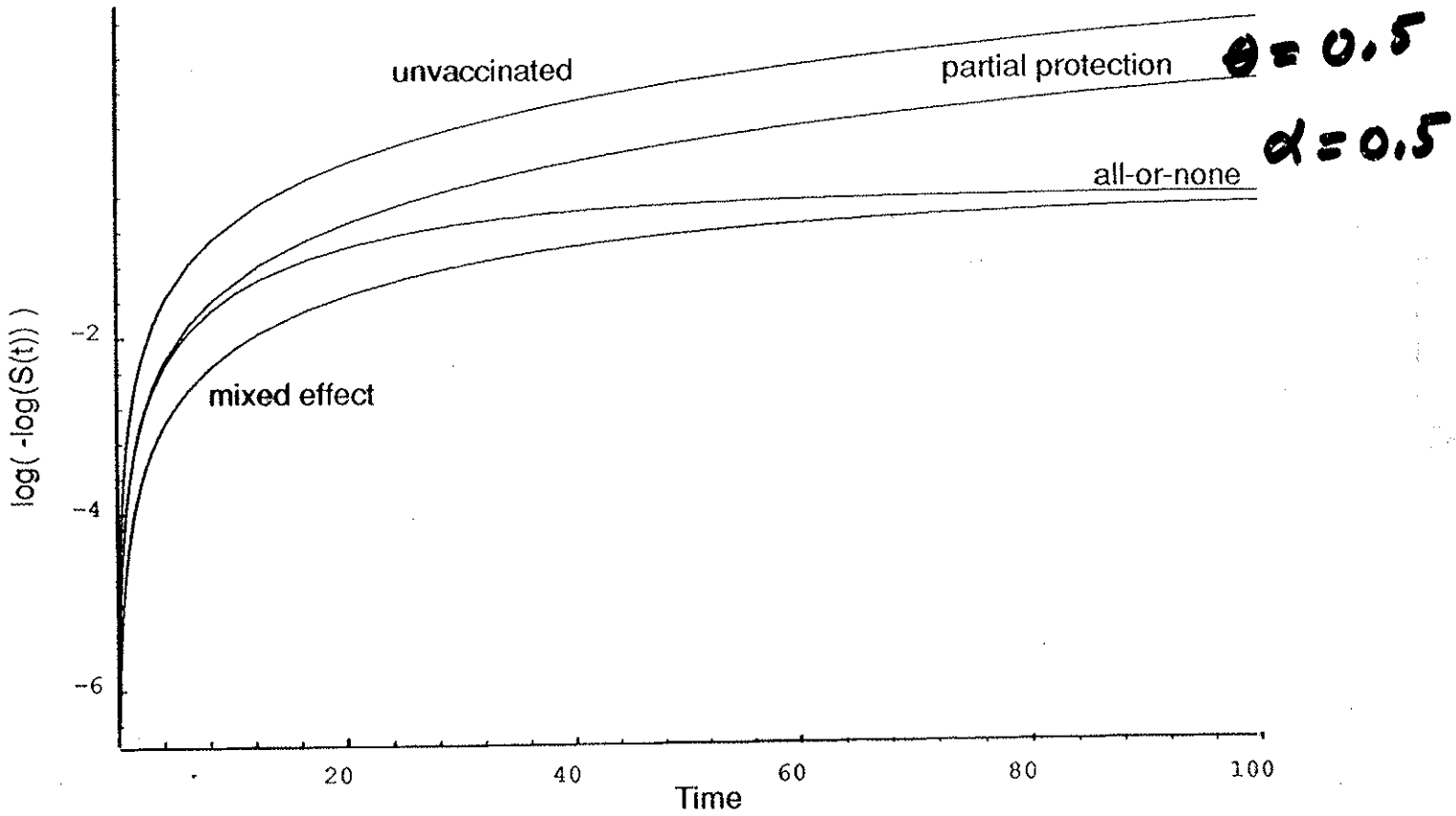
$$\ln\{-\ln[S_1(t)]\} = \ln[\Lambda_0(t)] + \ln(\theta)$$

e.g., pure all-or-none model ( $\delta_0 = \delta_1 = 0, \alpha_0 = 0, \alpha_1 = \alpha, \theta = 1$ )

$$S_0(t) = \exp[-\Lambda_0(t)]$$

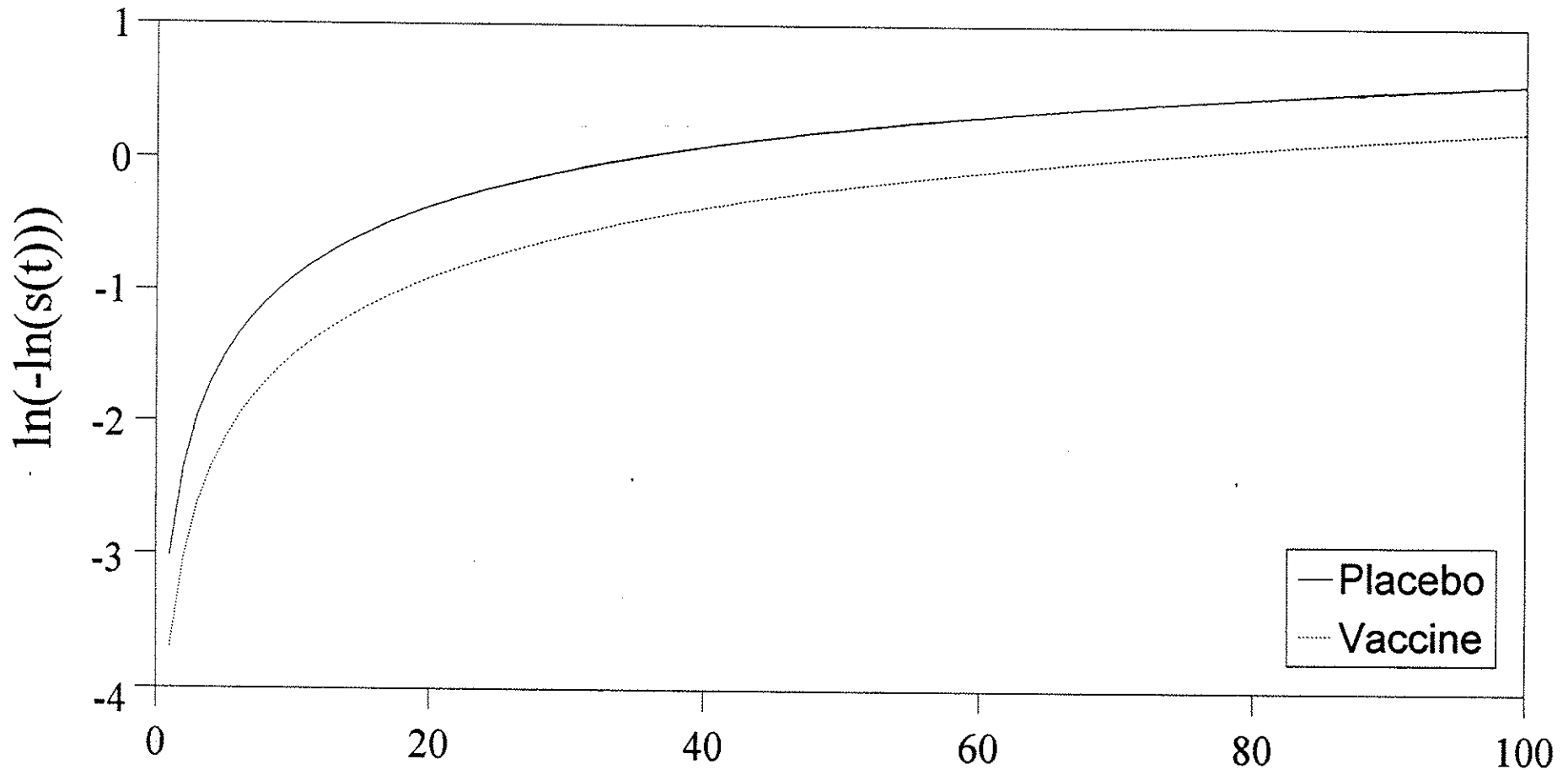
$$S_1(t) = \alpha + (1 - \alpha)\exp[-\Lambda_0(t)]$$





# Log-log Plot Of Leaky Model With Heterogeneity

$a=0.05, \alpha=0, \theta=0.5, \delta=1$

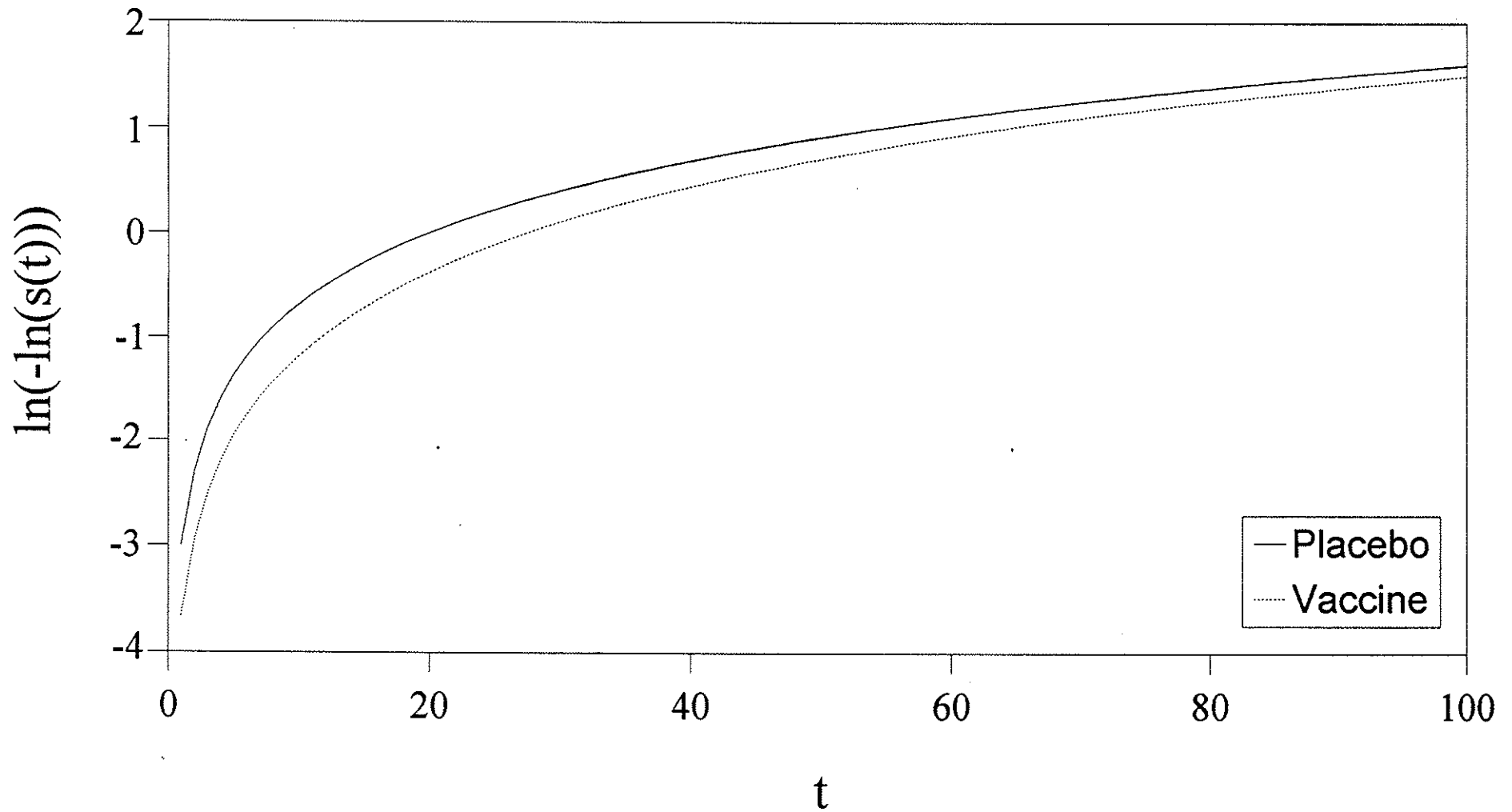


$$s_0(t) = \left[ \frac{1}{1 + at} \right]$$

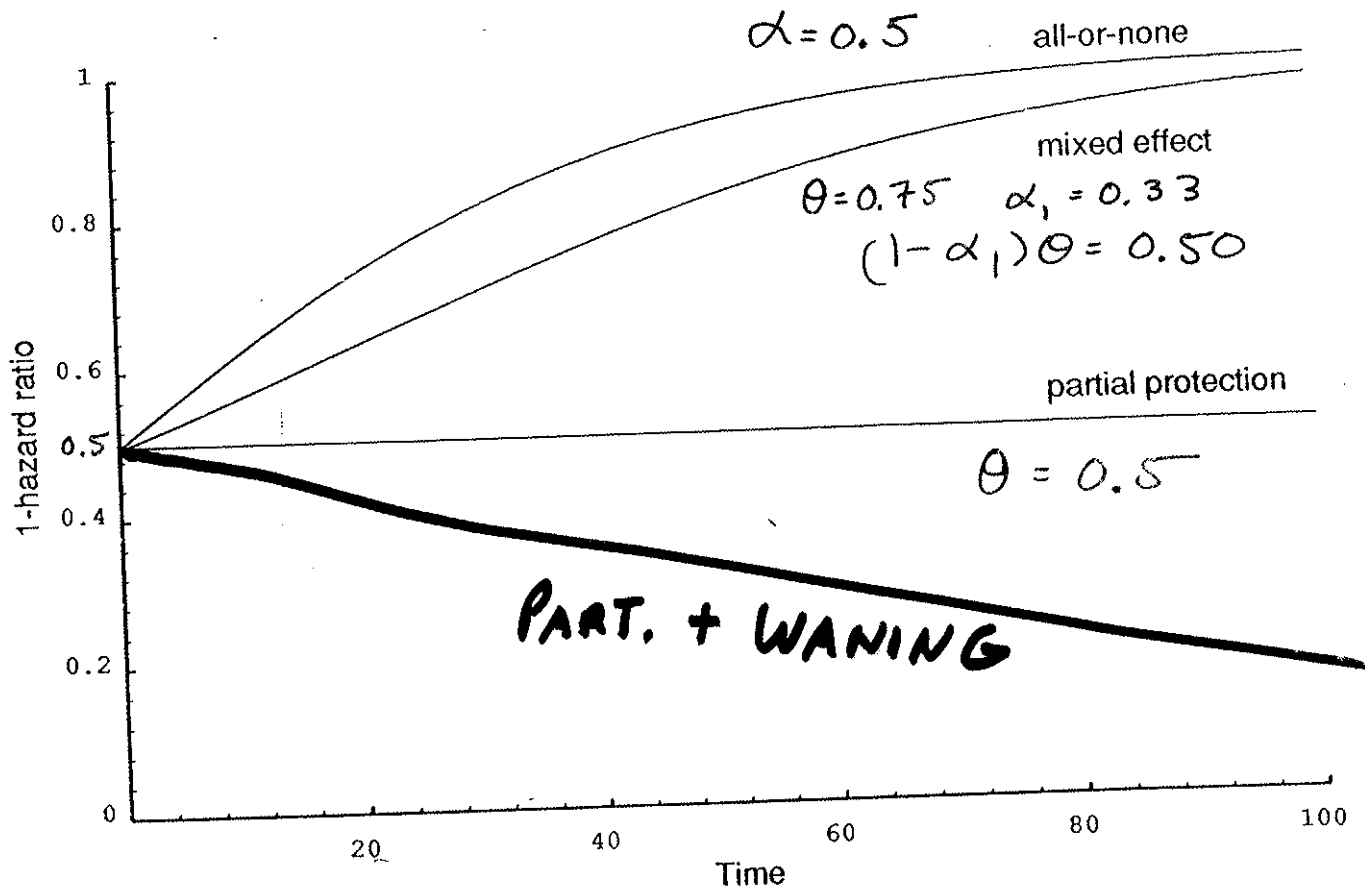
$$s_1(t) = \left[ \frac{1}{1 + a\theta t} \right]$$

# Log-log Plot of Leaky Model With Waning

$a = 0.05, \alpha = 0, \theta = 0.5, \delta = 0, \omega = 0.05$



$$1 - \frac{\lambda_1(t)}{\lambda_0(t)}$$



## STATISTICAL INFERENCE ON GROUPED DATA, $\phi = 1$

**Parameters:**  $\alpha_0, \alpha_1, \delta_0, \delta_1, \theta$ , and  $a = c\pi$

**Data:** Observations are made at times  $t_0 (=0), t_1, \dots, t_k$ .

Define the time intervals,  $[t_{i-1}, t_i)$ ,  $i = 1, \dots, k$ .

$p(t) = p_i$  in interval  $i$ ,

$$\Lambda_0(t) = c\pi \int_0^t p(\tau) d\tau = c\pi\kappa \left[ \sum_{j=1}^i (t_j - t_{j-1}) p_j + (t - t_i) p_i \right], \quad t \in [t_i, t_{i+1}).$$

$r_{iv}$  number at risk at beginning of  $i$

$m_{iv}$  number infected during  $i$

**Likelihood function:**

$$L = \prod_{i=1}^k \prod_{v=0}^1 \{S_v(t_i)/S_v(t_{i-1})\}^{(r_{iv} - m_{iv})} [1 - \{S_v(t_i)/S_v(t_{i-1})\}]^{m_{iv}},$$

**Table 1.**  
*Numbers at risk, ill, and monthly exposure for the measles epidemic  
 Muyinga, Burundi, April - November, 1988*

i	Month	Unvaccinated			Vaccinated			Exposure $p_i \times 100$ Percent
		At Risk*	Ill	Percent	At Risk	Ill	Percent	
1	April	579	10	1.7	857	9	1.1	1.3
2	May	551	13	2.4	848	13	1.5	1.9
3	June	517	10	1.9	835	2	0.2	0.9
4	July	483	12	2.5	833	20	2.4	2.4
5	Aug.	451	22	4.9	813	18	2.2	3.2
6	Sept.	408	50	12.3	795	24	3.0	6.4 <sup>†</sup>
7	Oct.	337	12	3.6	771	7	0.9	1.7
8	Nov.	317	0	0.0	764	0	0.0	0.0
<b>Total</b>			129			93		

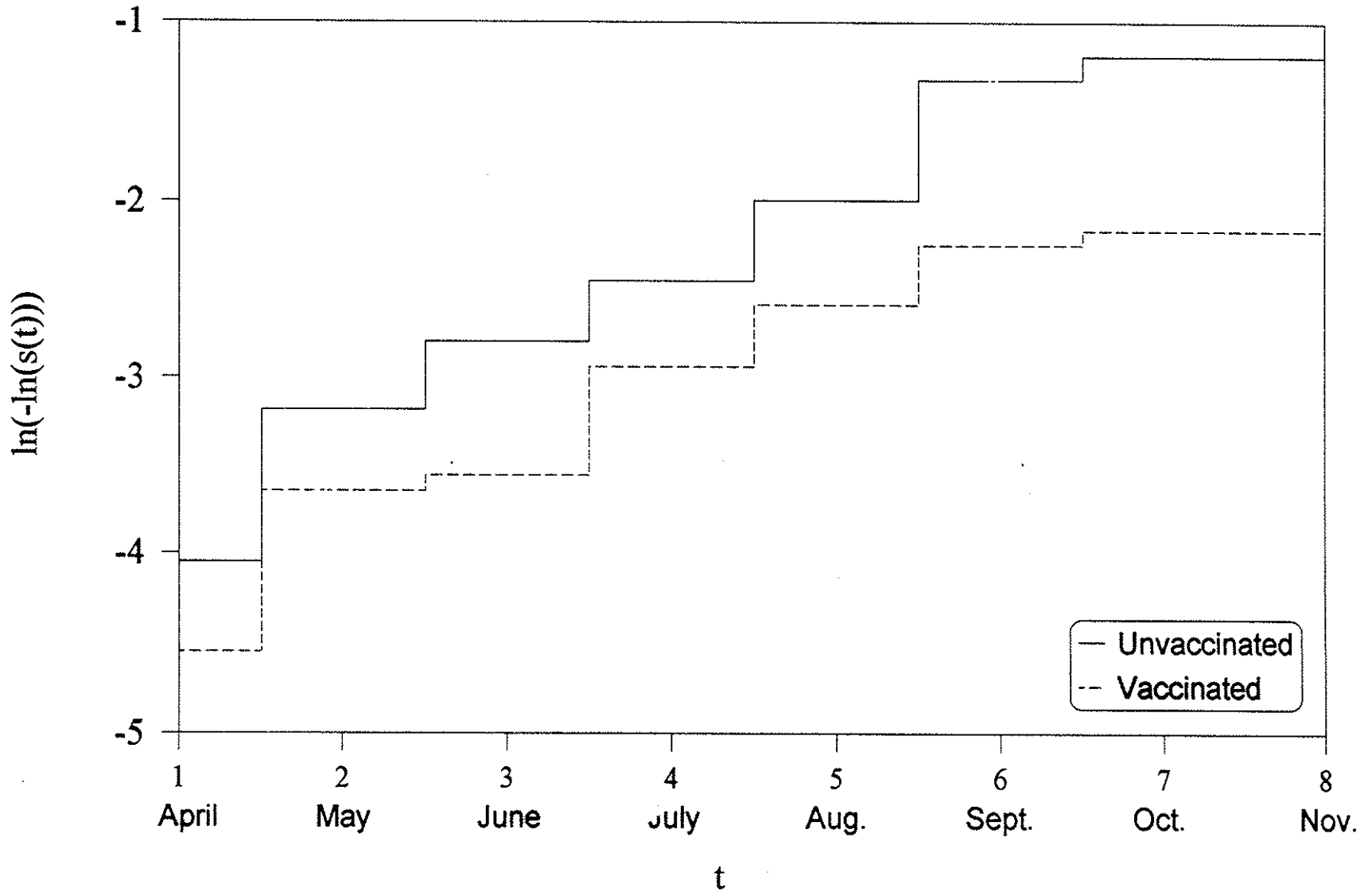
\* 140 initially at-risk unvaccinated children were vaccinated during the epidemic, and their vaccination times were treated as right censoring times for measles illness

† Includes three individuals who were vaccinated and who contracted measles in September. These individuals were treated as being unvaccinated with right-censored times for the purpose of estimation.



# Ln-ln Plot of Observed Data

## Burundi Measles Data



MEASLES OUTBREAK IN MUYINGA, BURUNDI, MARCH - DECEMBER, 1988 (CONTINUED)

$$m = 7, \alpha_0 = \delta_0 = \delta_1 = 0$$

**Estimates:**  $\hat{a} = c\pi\kappa = 1.658 \pm 0.137, \hat{\alpha}_1 = 0.805 \pm 0.060,$   
 $\hat{\theta} = 2.764 \pm 1.235$

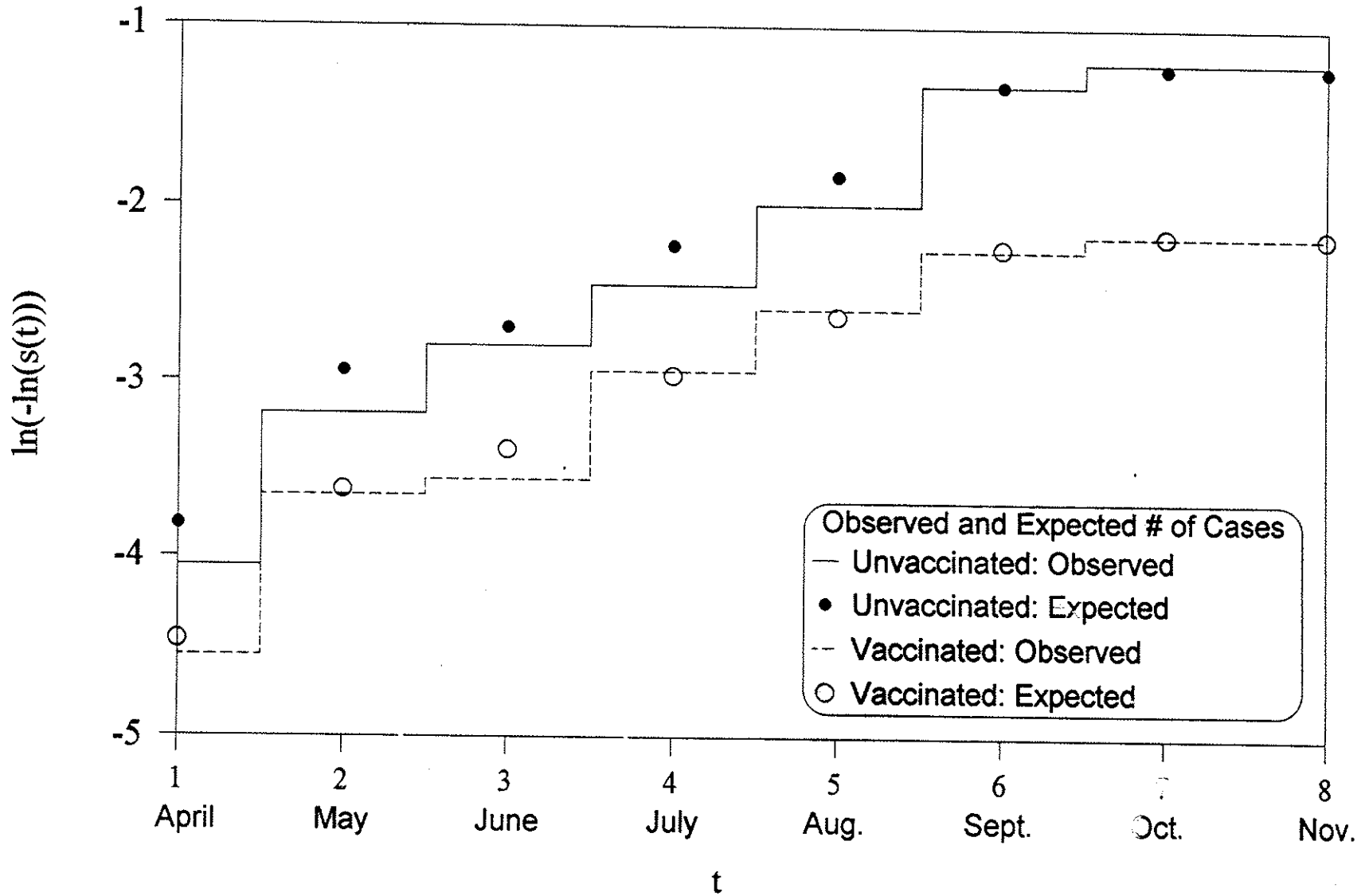
$$\hat{VE} \text{ all-or-none} = 0.805 \quad [0.687, 0.924]$$

$$\hat{VE} \text{ part.} = -1.765 \quad [-4.185, -0.657].$$

$$\hat{VE} \text{ gen.} = 0.462 \quad [0.318, 0.671]$$

# Ln-Ln Plot of Observed and Expected Data

## Burundi Measles Data



**Table 2.**  
*Observed and expected frequencies for the model fitted to the  
 data from the measles outbreak in Muyinga, Burundi,  
 April - October, 1988*

i	Month	Unvaccinated		Vaccinated	
		Observed	Expected	Observed	Expected
1	April	10	12.6	9	9.8
2	May	13	16.7	13	12.8
3	June	10	7.6	2	5.8
4	July	12	19.1	20	14.7
5	Aug.	22	23.0	18	16.7
6	Sept.	50	41.0	24	27.2
7	Oct.	12	9.4	7	6.0
Total		129	129.3	93	93.0

$\chi_{11}^2 = 12.8 \quad (p=0.3)$

Observed and Expected Number Ill Using  
 Degenerate Model with Estimates:  
 $a=1.98761$ ,  $\alpha=0.59442$ ,  $\theta=0.78174$

Month	Unvaccinated			Vaccinated*			p(t)
	At Risk	Obs. Ill	Exp. Ill	At Risk	Obs. Ill	Exp. Ill	
1	1865.0	30.	31.6901	490.0	3.	2.6448	.00862
2	1835.0	132.	155.3851	487.0	14.	13.0952	.04452
3	1703.0	167.	169.4726	473.0	13.	14.3584	.05274
4	1536.0	108.	92.5926	460.0	6.	8.0008	.03128
5	1428.0	90.	77.4975	454.0	9.	6.8868	.02807

Chi-Square=9.6139

\* Vaccinated with card

$t' = 6$

$p = 0.14$

MEASLES OUTBREAK IN CHAD, FEBRUARY - JUNE, 1993  
(CONTINUED)

$$m = 5, \alpha_0 = \delta_0 = \delta_1 = 0$$

$\swarrow$   $VE_{AN}$

**Estimates:**  $a = c\pi\kappa = 1.988 \pm 0.084$ ,  $\alpha_1 = 0.594 \pm 0.111$ ,  
 $(-VE_s = \theta = 0.782 \pm 0.238$

$$VE \text{ all-or-none} = 0.594 [0.378, 0.811]$$

$$VE \text{ part.} = 0.218 [-0.248, 0.684]$$

$$VE \text{ gen.} = 0.683 [0.504, 0.925]$$

**Table 2. Estimated vaccine efficacy using the summary model ( $VE_{SUM}$ ), partial protection model ( $VE_{PP}$ ) and all-or-none model ( $VE_{ALL}$ ) for data simulated with 10,000 people in both the vaccinated and unvaccinated groups, 60 time periods, 5% right censoring in the unvaccinated group, baseline hazard  $\lambda_u(t) = 0.05$  and  $\delta_0 = \delta_1 = 0$ .**

$\alpha_1^*$	Model	Point estimate and empirical 95% confidence interval for vaccine efficacy <sup>†</sup>			
		$1-\theta^{\ddagger} = 0.2$	$1-\theta = 0.4$	$1-\theta = 0.6$	$1-\theta = 0.8$
0.2	Preset <sup>§</sup>	0.36	0.52	0.68	0.84
	$VE_{SUM}$	0.36 (0.34-0.38) <sup>†</sup>	0.52 (0.50-0.54)	0.68 (0.67-0.69)	0.84 (0.83-0.85)
	$VE_{PP}$	0.52 (0.50-0.53)	0.61 (0.60-0.62)	0.72 (0.71-0.73)	0.85 (0.84-0.86)
	$VE_{ALL}$	0.23 (0.22-0.24)	0.29 (0.28-0.30)	0.40 (0.39-0.41)	0.61 (0.60-0.62)
0.4	Preset	0.52	0.64	0.76	0.88
	$VE_{SUM}$	0.52 (0.50-0.54)	0.64 (0.62-0.66)	0.76 (0.75-0.77)	0.88 (0.87-0.89)
	$VE_{PP}$	0.71 (0.70-0.72)	0.75 (0.74-0.76)	0.81 (0.81-0.82)	0.89 (0.89-0.90)
	$VE_{ALL}$	0.42 (0.41-0.43)	0.47 (0.46-0.48)	0.55 (0.54-0.56)	0.71 (0.70-0.72)
0.6	Preset	0.68	0.76	0.84	0.92
	$VE_{SUM}$	0.68 (0.66-0.70)	0.76 (0.74-0.77)	0.84 (0.83-0.85)	0.92 (0.91-0.93)
	$VE_{PP}$	0.84 (0.83-0.84)	0.86 (0.85-0.86)	0.89 (0.88-0.89)	0.93 (0.93-0.94)
	$VE_{ALL}$	0.62 (0.61-0.63)	0.65 (0.64-0.66)	0.70 (0.69-0.71)	0.81 (0.80-0.82)
0.8	Preset	0.84	0.88	0.92	0.96
	$VE_{SUM}$	0.84 (0.83-0.85)	0.88 (0.87-0.89)	0.92 (0.91-0.93)	0.96 (0.96-0.96)
	$VE_{PP}$	0.93 (0.93-0.93)	0.94 (0.93-0.94)	0.95 (0.95-0.95)	0.97 (0.97-0.97)
	$VE_{ALL}$	0.81 (0.80-0.82)	0.82 (0.82-0.83)	0.85 (0.85-0.86)	0.91 (0.90-0.91)

\*  $\alpha_1$  = proportion completely protected in vaccinated group.

<sup>†</sup> Average point estimate based on 1,000 simulations per  $\alpha_1$ ,  $1-\theta$  combination.

<sup>‡</sup>  $\theta$  = relative residual susceptibility of vaccinated susceptibles compared to the unvaccinated group.

<sup>§</sup> Preset value of  $VE_{SUM}$  in the simulation model =  $1-(1-\alpha_1)\theta$ .

<sup>†</sup> Empirical 95% confidence intervals based on 1,000 simulations per  $\alpha_1$ ,  $1-\theta$  combination.

# Cholera Vaccines

Durham, L.K., Longini, I.M., Halloran, M.E., Clemens, J.D., Nizam, A. and Rao, M.: Estimation of vaccine efficacy in the presence of waning: Application to cholera vaccines. *American Journal of Epidemiology* **147**, 948-959 (1998).

Durham, L.K., Halloran, M.E., Longini, I.M. and Manatunga, A.K.: Comparing two smoothing methods for exploring waning vaccine effects. *Applied Statistics* **48**, 395-407 (1999).

Longini, I.M., Nizam, A., Ali, M., Yunus, M., Shenvi, N. and Clemens, J.D.: Controlling endemic cholera with oral vaccines. *Public Library of Science (PloS), Medicine* **4** (11) 2007: e336  
[doi:10.1371/journal.pmed.0040336](https://doi.org/10.1371/journal.pmed.0040336).

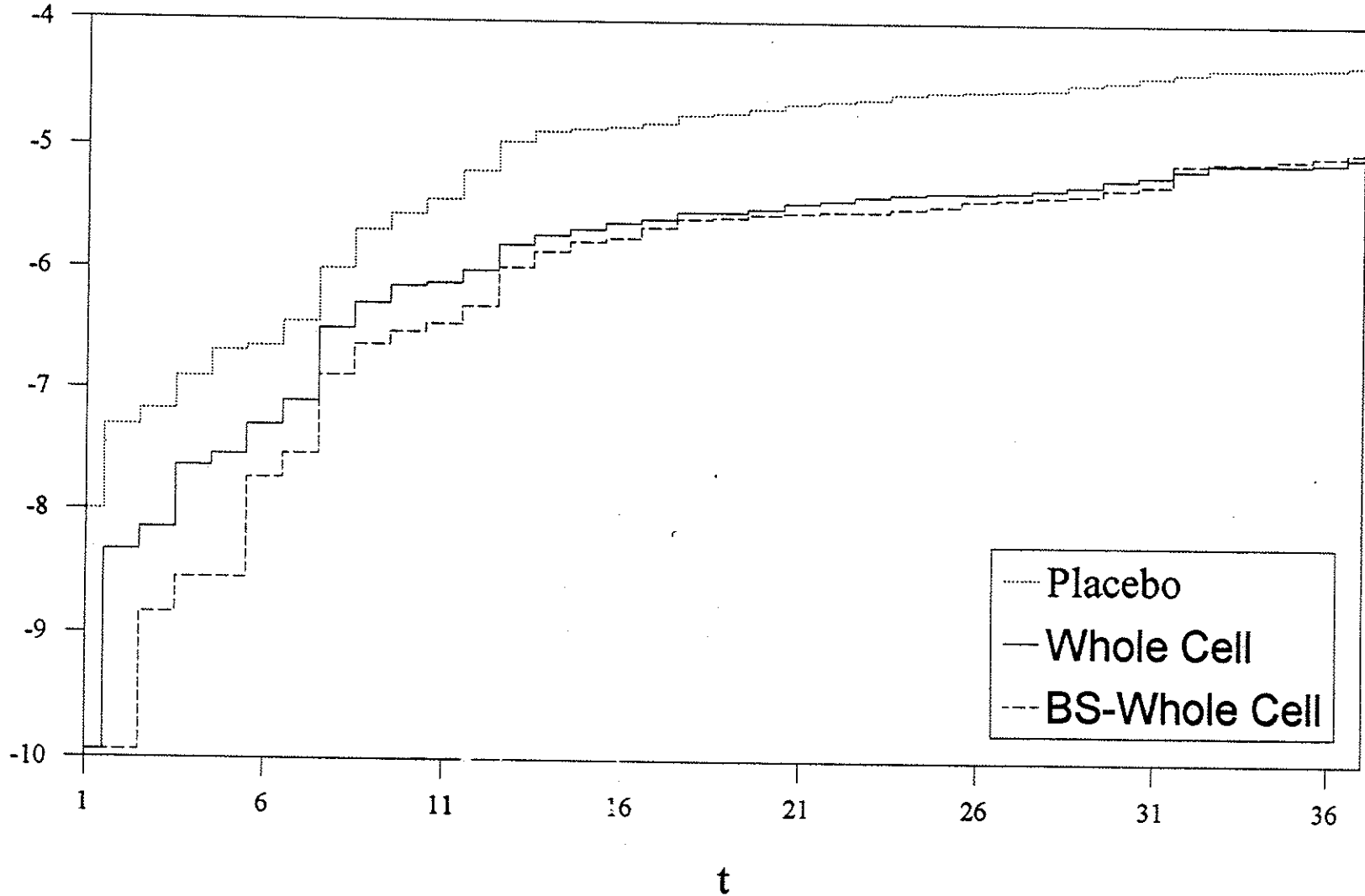


**Table 1**  
*Data from the Bangladesh cholera vaccine trial (Clemens, et al., 1990)*

Month i	Vaccine Group									
	Placebo v=0			WC v=1			BS-WC v=2			P <sub>i</sub>
	At Risk r <sub>0i</sub>	Ill d <sub>0i</sub>	Lost	At Risk r <sub>1i</sub>	Ill d <sub>1i</sub>	Lost	At Risk r <sub>2i</sub>	Ill d <sub>2i</sub>	Lost	
0 Initial	20837			20743			20705			
1 May '85	20822.0	7	30	20723.5	1	39	20689.5	1	31	0.00014
2 Jun	20777.0	7	46	20681.5	4	43	20649.0	0	48	0.00032
3 Jul	20722.0	2	50	20633.5	1	45	20600.0	2	50	0.00040
4 Aug	20677.0	5	36	20589.0	4	42	20547.0	1	52	0.00056
5 Sep	20612.5	5	83	20530.0	1	58	20485.0	0	70	0.00066
6 Oct	20536.5	1	59	20466.0	3	58	20423.0	5	54	0.00081
7 Nov	20484.5	6	43	20414.5	3	39	20374.0	2	34	0.00099
8 Dec	20442.5	18	29	20375.5	14	33	20337.5	10	35	0.00167
9 Jan '86	20360.5	19	99	20307.0	7	76	20265.5	6	89	0.00220
10 Feb	20252.5	10	79	20219.0	6	86	20192.0	3	46	0.00251
11 Mar	20165.5	10	75	20141.0	1	58	20135.5	2	61	0.00273
12 Apr	20090.0	23	56	20080.5	5	61	20075.0	5	56	0.00328
13 May	20020.5	30	37	20023.5	12	43	20021.0	14	42	0.00421
14 Jun	19937.0	12	70	19960.5	5	59	19953.0	7	66	0.00461
15 Jul	19860.0	3	60	19896.5	3	59	19883.5	5	59	0.00479
16 Aug	19791.5	3	71	19836.5	4	55	19815.5	2	67	0.00494
17 Sep	19732.0	5	42	19785.5	2	39	19752.5	6	55	0.00516
18 Oct	19689.5	11	33	19744.0	5	40	19702.0	5	34	0.00552
19 Nov	19650.5	3	23	19702.5	0	33	19663.5	1	32	0.00559
20 Dec	19614.0	6	44	19671.0	2	30	19626.5	2	39	0.00576
21 Jan '87	19545.0	8	82	19620.5	4	67	19566.0	1	78	0.00598
22 Feb	19464.5	4	63	19552.0	2	62	19498.0	1	56	0.00610
23 Mar	19403.0	3	52	19486.0	3	66	19436.5	0	65	0.00620
24 Apr	19349.5	9	49	19433.5	2	33	19385.0	2	38	0.00642
25 May	19290.0	3	52	19393.0	1	44	19346.5	2	35	0.00653
26 Jun	19233.5	2	55	19348.5	0	43	19299.0	4	56	0.00663
27 Jul	19175.5	2	57	19295.5	1	63	19235.5	1	63	0.00670
28 Aug	19099.5	2	91	19206.0	2	14	19146.0	2	114	0.00680
29 Sep	19028.0	9	48	19118.0	3	58	19054.0	1	66	0.00703
30 Oct	18969.5	5	51	19063.5	5	45	18997.0	5	46	0.00730
31 Nov	18912.5	9	53	19013.0	3	46	18950.0	3	38	0.00756
32 Dec	18863.0	7	28	18972.5	6	29	18909.0	18	38	0.00811
33 Jan '88	18824.0	9	36	18933.0	5	38	18854.5	2	35	0.00839
34 Feb	18786.5	0	21	18900.0	0	18	18822.0	0	26	0.00839
35 Mar	18766.0	1	20	18882.0	0	18	18793.5	3	31	0.00846
36 Apr	18749.5	2	11	18866.5	2	13	18771.5	3	7	0.00858
37 May	18739.0	5	6	18852.0	5	12	18761.5	4	7	0.00883

# Ln-Ln Plot: Observed Bangladesh Cholera Data

$$\ln(-\ln(s(t))) = \ln(-\ln(I(t)))$$



BA671

DURHAM, HALLORAN, LONGINI

1. SCHOENFELD RESIDUALS
2. GEN. ADD. MODELS (GAM)

$$\hat{V}_E(t) = 1 - \hat{R}_R(t) = 1 - e^{\hat{\beta}(t)}$$

SCHOENFELD RESIDUALS

$N_i(t)$  - COUNTING PROCESS

$\lambda_0(t)$  - BASELINE INTENSITY

INTENSITY FOR TIME INVARIANT MOD.

$$Y_i(t) e^{\beta' \mathbf{z}_i} \lambda_0(t)$$

FIT AND GET ESTIMATE OF  $\beta$

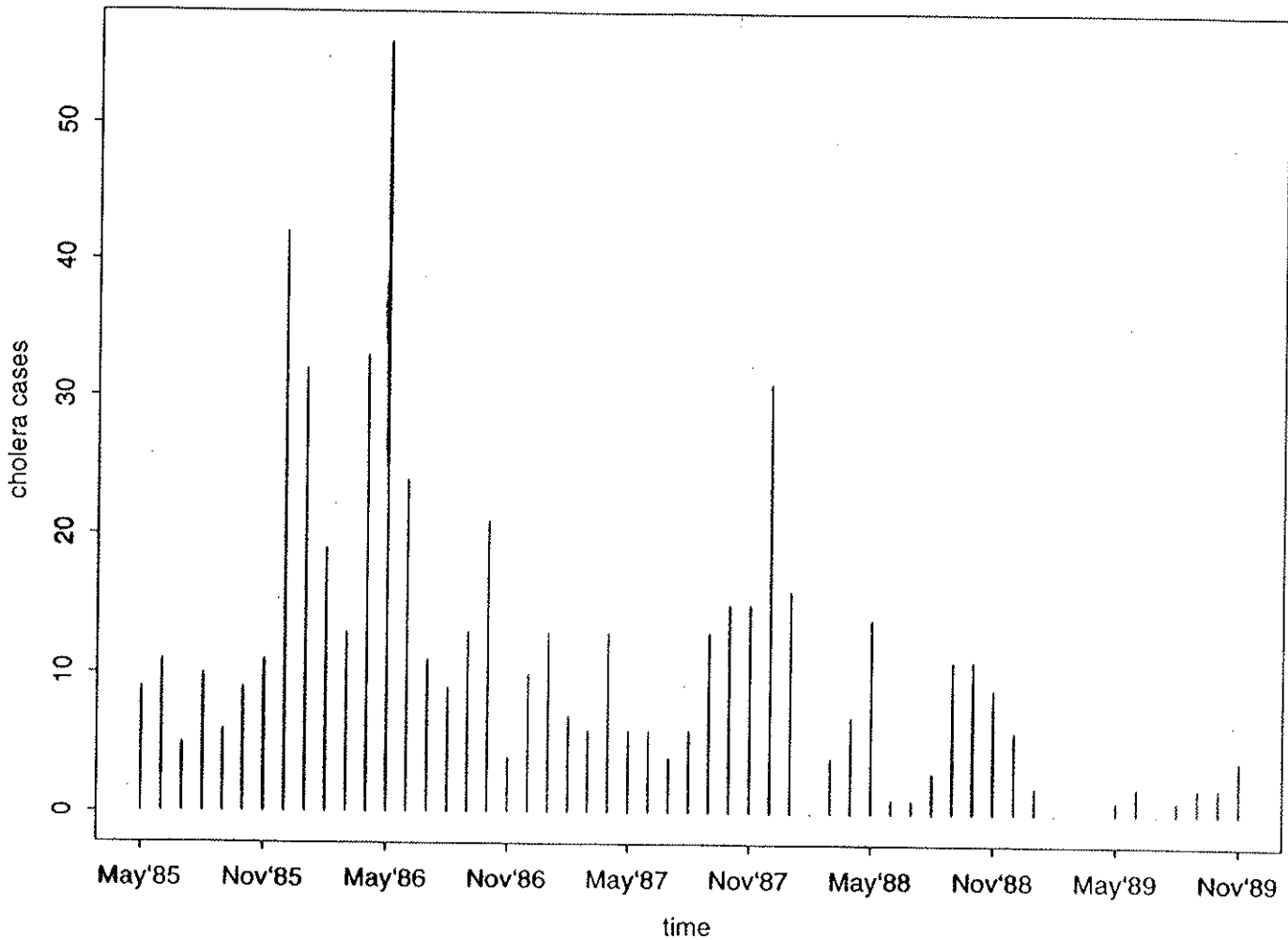
COMPUTE RESIDUAL AT FAILURE  
TIMES AND CONSTRUCT

$$\hat{\beta}(t) = \hat{\beta} + \hat{\theta}(t)$$

WHICH IS DEFINED AT EACH  
FAILURE TIME

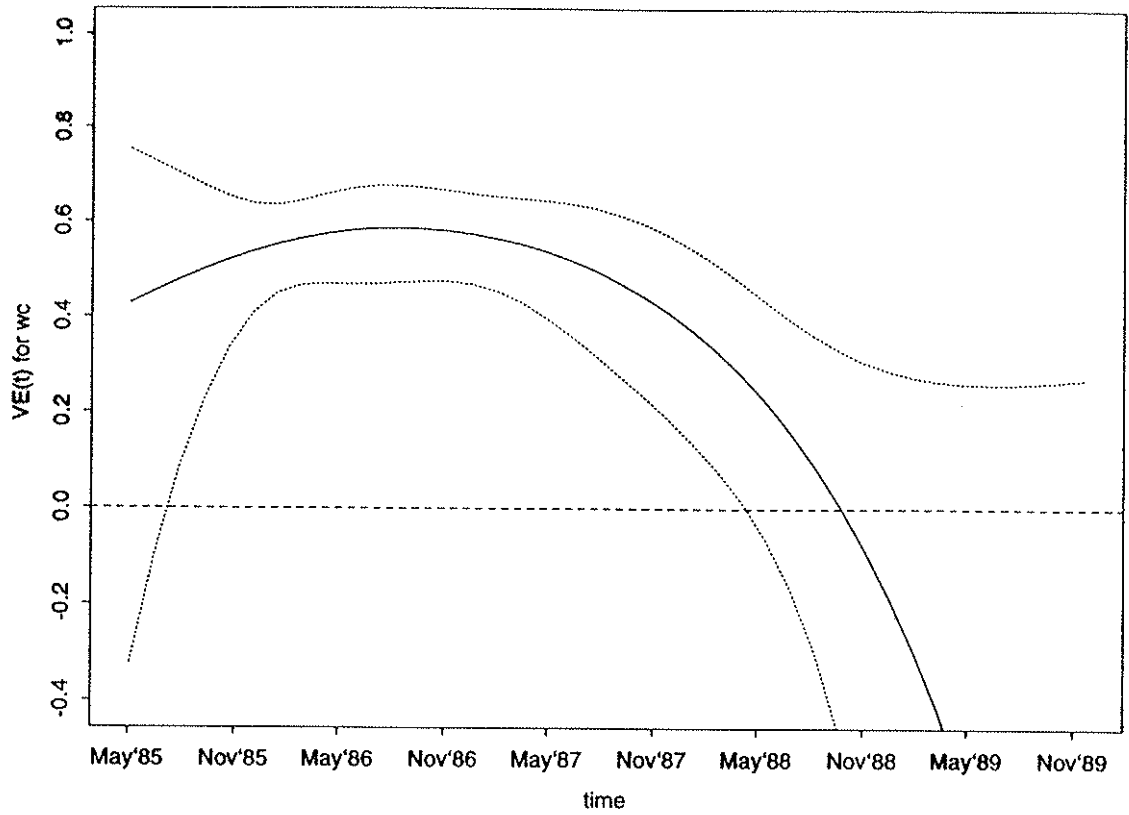
THEN SMOOTH  $\hat{\beta}(t)$

number of new cholera cases per month



**580 CASES**

### WC vaccine



### BS-WC vaccine

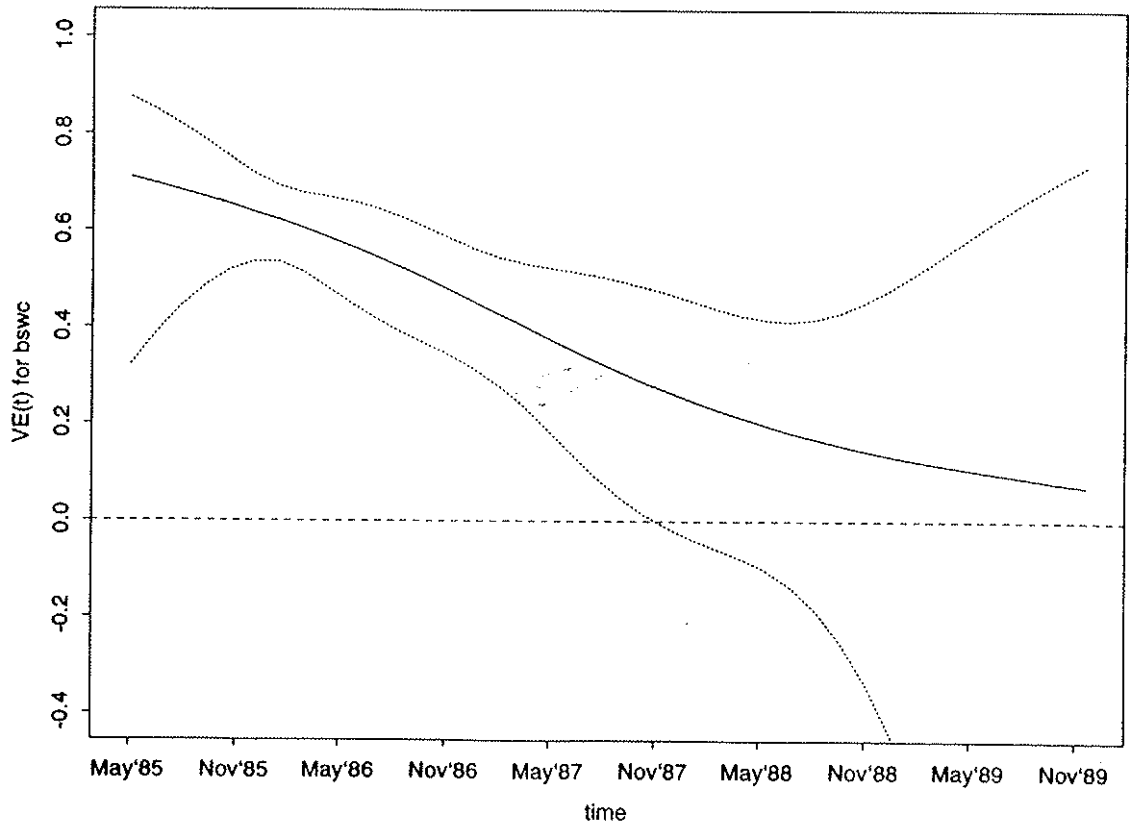


Table 3: Estimates of  $VE(t)$ , with 95% confidence intervals, for the WC and BS-WC vaccines, Matlab, Bangladesh, May 1, 1985-November 31, 1989

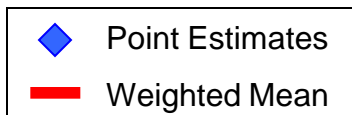
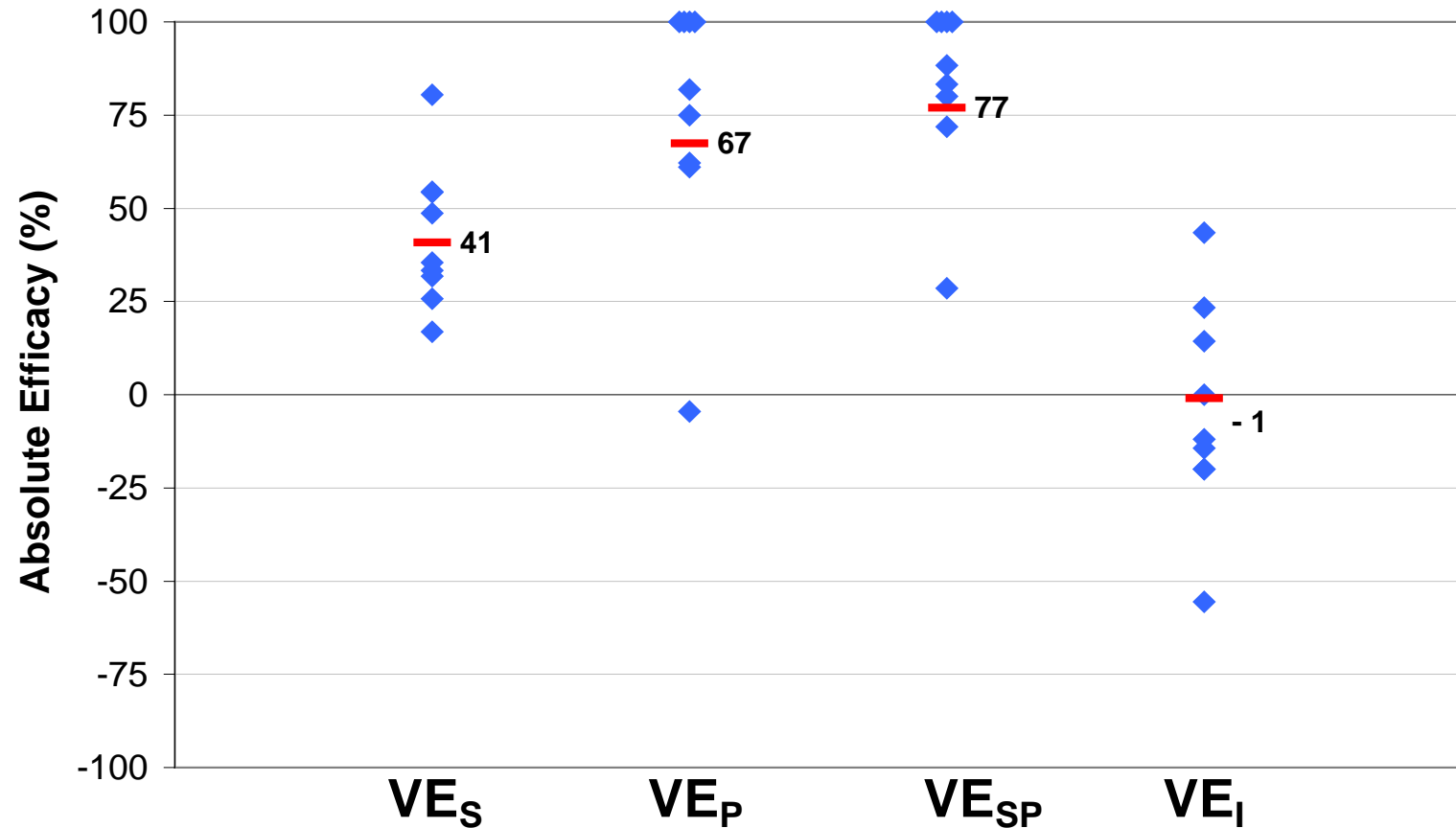
date	day	whole cell		B-subunit whole cell	
		VE(day)	approx. 95% c.i.	VE(day)	approx. 95% c.i.
May 85	0	0.430	(-0.342, 0.758)	0.713	(0.320, 0.879)
Nov 85	183	0.525	(0.356, 0.650)	0.650	(0.523, 0.743)
May 86	365	0.579	(0.467, 0.667)	0.572	(0.457, 0.662)
Nov 86	548	0.583	(0.478, 0.667)	0.476	(0.344, 0.582)
May 87	730	0.538	(0.394, 0.648)	0.374	(0.176, 0.524)
Nov 87	913	0.433	(0.220, 0.588)	0.280	(0.006, 0.478)
May 88	1095	0.245	(-0.028, 0.445)	0.202	(-0.089, 0.416)
Nov 88	1278	-0.073	(-0.664, 0.308)	0.141	(-0.338, 0.448)
May 89	1460	-0.593	(-2.40, 0.257)	0.092	(-0.955, 0.578)

# Influenza Challenge Studies

- Clements ML et al. Advantage of live attenuated cold-adapted influenza A virus over inactivated vaccine for A/Washington/80 (H3N2) wild-type virus infection. *Lancet* 1984;1:705-8.
- Clements ML et al. Resistance of adults to challenge with influenza A wild-type virus after receiving live or inactivated virus vaccine. *J Clin.Microbiol.* 1986;23:73-6.
- Sears SD et al. Comparison of live, attenuated H1N1 and H3N2 cold-adapted and avian-human influenza A reassortant viruses and inactivated virus vaccine in adults. *J Infect.Dis.* 1988;158:1209-19.
- Clements ML et al. Evaluation of the infectivity, immunogenicity, and efficacy of live cold-adapted influenza B/Ann Arbor/1/86 reassortant virus vaccine in adult volunteers. *J Infect.Dis.* 1990;161:869-77.
- Treanor JJ et al. Evaluation of trivalent, live, cold-adapted (CAIV-T) and inactivated (TIV) influenza vaccines in prevention of virus infection and illness following challenge of adults with wild-type influenza A (H1N1), A (H3N2), and B viruses. *Vaccine* 1999;18:899-906.

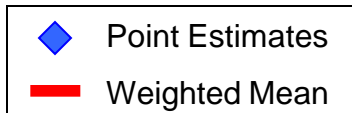
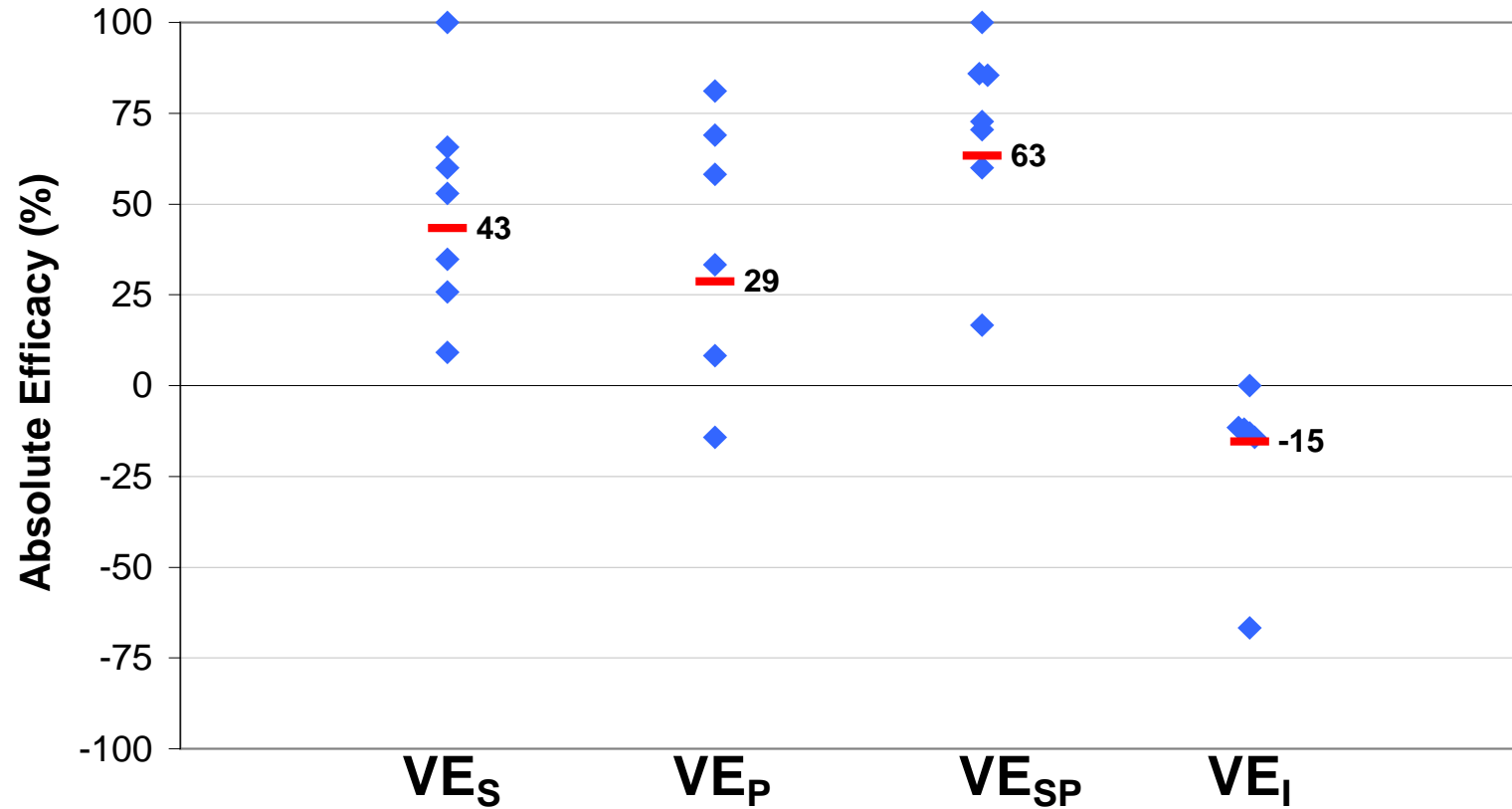


## Absolute Efficacy of Live Influenza Vaccine



Source: Basta NE, Halloran ME, Matrajt L, Longini IM. *AJE* (2008)

## Absolute Efficacy of Inactivated Influenza Vaccine



Source: Basta NE, Halloran ME, Matrajt L, Longini IM. *AJE* (2008)