Math 575 Syllabus

Textbooks: The following suggested syllabus is based on *Advanced Calculus* (Second Edition) by Patrick M. Fitzpatrick and *Principles of Mathematical Analysis* (Third Edition) by Walter Rudin. References to Fitzpatrick's book are indicated by [F], and those to Rudin' book by [R].

1. Continuity Revisited ([F] §3.7, 9.2, 9.3 & Theorem (9.31)) [4 lectures]
Quick review of continuity: ϵ - δ definition and characterization using sequences
Limits: $\lim_{x\to a} f(x)$, ϵ - δ definition, characterization using sequences, properties, and the corresponding results for continuous functions as corollaries ([F] §3.7)
Pointwise convergence, uniform convergence, examples, and uniformly Cauchy sequences ([F] §9.2, 9.3)
Uniform limit of continuous functions is continuous ([F] Theorem (9.31))

2. Differentiation ([F] Ch. 4 & [R] Ch. 5) [6 - 7 lectures]
Definition of the derivative, examples: $x^n, |x|$
Tangent line approximation, necessity of continuity
Algebra of derivatives, chain rule, examples: polynomials, x^{-n}
Derivatives of inverses of strictly monotone functions, examples: $x^{\frac{1}{n}}, x^{\frac{m}{n}}$
Rolle's theorem, Mean Value Theorem and its consequences, Cauchy MVT
Higher derivatives and Taylor's theorem
Intermediate Value Theorem for derivatives

3. The Riemann-Stieltjes Integral ([R] p. 120-134 & [F] Theorem (6.26)) [8 - 9 lectures]
Upper and lower sums, definition of the Riemann-Stieltjes integral
Integrability of continuous functions; integrability of monotone functions (for continuous α)
Properties of the integral
Change of variable
Fundamental Theorem of Calculus, integration by parts
Mean Value Theorem for Riemann integrals

4. Uniform Convergence and Power Series ([R] Theorems (7.16), (7.17), (7.10), (8.1)) [5 lectures]
Uniform convergence and integration
Uniform convergence and differentiation
Weierstrass' M-test
Power series: radius of convergence, examples, absolute and uniform convergence
Differentiation of power series
(Time permitting: Weierstrassn Approximation Theorem ([F] §8.7))