

Math 574 Syllabus

Textbooks: *Advanced Calculus* (Second Edition) by Patrick M. Fitzpatrick and *Principles of Mathematical Analysis* (Third Edition) by Walter Rudin. Fitzpatrick is the main text. In the following, references to Rudin's book are indicated by [R]; references without this indication are to Fitzpatrick's book.

1. The Real Number System (Ch. 1 & [R] p. 1-10.) [5 - 6 lectures]

Quick review of sets, functions, equivalence relations, \mathbb{N} , \mathbb{Z} and \mathbb{Q}

Fields, ordered fields, and their properties

There is no rational number x such that $x^2 = 2$. (Usual classical proof.)

The least upper bound property, \mathbb{R} , suprema and infima

The Archimedean property

Intervals, absolute value and the triangle inequality

Density of \mathbb{Q} in \mathbb{R}

For $c > 0$ and $n \in \mathbb{N}$, there exists a unique $x \in \mathbb{R}$ such that $x^n = c$. (Prove $x = \sup\{r : r^n < c\}$.)

2. Sequences (§2.1 - §2.4, part of §9.1, [R] p. 56-58.) [8 lectures]

Sequences and limits: basic definitions and facts (§2.1, and Theorems (2.18) and (2.19))

Sandwich (squeeze) theorem

Standard sequences: $\frac{1}{n^p}$, $c^{\frac{1}{n}}$, $n^{\frac{1}{n}}$ ([R] Theorem (3.20) (a) - (c))

Monotone sequences (§2.3)

Subsequences and sequential compactness (§2.4)

\liminf and \limsup : define, and prove \liminf equals \limsup (as a finite number) iff limit exists

Cauchy sequences, and completeness of \mathbb{R} (§9.1 through Theorem (9.4))

3. Continuous Functions (§3.5, §3.1 - §3.3, §3.6, plus uniform continuity) [6 lectures]

ϵ - δ definition of continuity, characterization using sequences, algebra, compositions (§3.5 and §3.1)

Extreme Value Theorem (§3.2)

Intermediate Value Theorem (§3.3)

Monotone functions: continuity and inverses (§3.6)

Uniform Continuity: ϵ - δ definition of uniform continuity, continuous $f : [a, b] \rightarrow \mathbb{R}$ is uniformly continuous. (You can use contradiction and repeated bisection of the domain to prove this. Skip the definition of uniform continuity given in §3.4, and instruct students to do so as well.)

4. Series (§9.1 & parts of [R] Ch. 3) [5 - 6 lectures]

Convergence and Cauchy criterion for series, absolute convergence

Series of non-negative terms: comparison test, Cauchy Condensation Theorem ([R] Theorem (3.27))

Standard series: $\sum x^n$, $\sum \frac{1}{n}$, $\sum \frac{1}{n^p}$ (Do not cover the integral test, use CCT instead.)

Root test and ratio test ([R] p. 65-69)

Alternating series, $\sum (-1)^n \frac{1}{n}$, re-arrangements ([R] Example (3.53))

Re-arrangements of absolutely convergent series ([R] Theorem (3.55))

(Time permitting: Cauchy product of series ([R] Theorem (3.50))